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VARIATION OF THE BINDING ENERGIES OF NEUTRONS AND PROTONS IN HEAVY NUCLII

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ABSTRACT. The binding energy of the last neutron and of the last proton for different $N-Z$) values has been calculated from $_{80}Hg^{202}$ to $_{100}$ centurium^{255} from current mass data The variation of binding energy has been studied and graphs have been plotted throughout this range. It has been observed from such studies that there are possibilities for the existence of magic numbers after 82 protons and 126 neutrons.

INTRODUCTION

According to the shell structure of the nucleus there are certain neutron and proton numbers which are called magic. Several nuclear properties show remarkable fluctuations around these numbers. The interest in shell structure began with a review by Mayer (1948) of the experimental information correlating abundance and stability with the higher magic numbers. Peaks in the binding energies for both neutrons and protons in light nuclei were observed at these numbers by Redlich (1952).

The masses of heavier nuclei and their isotopes were not known with sufficient accuracy to permit reliable calculation and to draw specific conclusions. Further, the number of isotopes known were small in number and masses of some of them were not determined. Since a number of data on masses from $_{80}Hg^{202}$ to $_{100}$ centurium^{255} have recently been reported by Segre (1953), Green (1955) and Duckworth, Banjamin and Pennington (1954), it was thought worthwhile to calculate and study the binding energies of neutrons and protons for heavy nuclei. The recent values of masses of neutron and neutral hydrogen atom as given by Li, Whaling, Fowler, and Lauritsen (1951) have also been taken into account for this purpose.

CALCULATION AND DESCRIPTION

The binding energy of the last neutron and of the last proton was calculated by the following relations:

$$B_n(A, N, Z) = M(A-1, N-1, Z)+m_n - M(A, N, Z)$$
$$B_p(A, N, Z) = M(A-1, N, Z-1)+m_H - M(A, N, Z)$$

where, B_n = Binding energy of last neutron
B_p = Binding energy of last proton
A = Mass number
N = Number of neutrons
Z = Atomic number or number of protons
M = Isotopic mass
m_n = Mass of the neutron = 1.008982 m.u.
m_H = Mass of the neutral hydrogen Atom = 1.008142 m.u.

The binding energies, thus calculated are given in Table 1.

TABLE 1

Element	(Z)	(A)	(N)	($N-Z$)	(M) m.u.	(B_n) m.m.u.	(B_p) m.m.u.
Hg	80	203	123	43	203.03550	..	..
		205	125	45	205.03980	..	..
Tl	81	203	122	41	203.03499	..	..
		204	123	42	204.03679	7.002	6.672
		205	124	43	205.03792	8.032	..
		206	125	44	206.04021	6.692	7.732
		207	126	45	207.04189	7.302	..
		208	127	46	208.04676	4.112	..
		209	128	47	209.05044	5.302	
		210	129	48	210.05537	4.052	..
Pb	82	204	122	40	204.03612	..	7.012
		205	123	41	205.03831	6.792	6.802
		206	124	42	206.03859	8.702	7.472
		207	125	43	207.04034	7.232	8.012
		208	126	44	208.04140	7.922	8.632
		209	127	45	209.04623	4.152	8.672
		210	128	46	210.04958	5.632	9.002
		211	129	47	211.05450	4.062	9.012
		212	130	48	212.05791	5.572	....
		213	131	49	213.06268	4.212	..
		214	132	50	214.06633	5.332	..

TABLE I (*contd.*)

Element	(*Z*)	(*A*)	(*N*)	(*N*−*Z*)	(*M*) m.u.	(B_n) m.m.u.	(B_p) m.m.u.
Bi	83	207	124	41	207.04285	..	3.882
		208	125	42	208.04451	7.322	3.972
		209	126	43	209.04550	7.992	4.042
		210	127	44	210.04951	4.972	4.862
		211	128	45	211.05300	5.492	4.722
		212	129	46	212.05728	4.702	5.362
		213	130	47	213.06072	5.542	5.332
		214	131	48	214.06526	4.442	5.562
		215	132	49	215.06739	6.852	7.082
Po	84	208	124	40	208.04558	..	5.412
		209	125	41	209.04750	7.062	5.152
		210	126	42	210.04826	8.222	5.382
		211	127	43	211.05234	4.902	5.312
		212	128	44	212.05487	6.452	6.272
		213	129	45	213.05922	4.632	6.202
		214	130	46	214.06185	6.352	7.012
		215	131	47	215.06643	4.402	6.972
		216	132	48	216.06919	6.222	6.342
		217	133	49	217.07354	4.632	..
		218	134	50	218.07676	5.762	..
At	85	211	126	41	211.05317	..	3.232
		212	127	42	212.05675	5.402	3.732
		213	128	43	213.05925	6.482	3.762
		214	129	44	214.06299	5.242	4.372
		215	130	45	215.06562	6.352	4.372
		216	131	46	216.06967	4.932	4.902
		217	132	47	217.07225	6.402	5.082
		218	133	48	218.07638	4.852	5.302
		219	134	49	219.07865	6.712	6.252
Em	86	212	126	40	212.05621	..	5.102
		215	129	43	215.06562	..	5.512
		216	130	44	216.06750	7.102	6.262
		217	131	45	217.07155	4.932	6.262

TABLE I (*contd.*)

Element	(Z)	(A)	(N)	($N-Z$)	(M) m.u.	(B_n) m.m.u.	(B_p) m.m.u.
Em	86	218	132	46	218.07351	7.022	6.882
		219	133	47	219.07776	4.732	6.762
		220	134	48	220.07993	6.812	6.862
		221	135	49	221.08385	5.062	..
		222	136	50	222.08663	6.202	..
Fr	87	217	130	43	217.07221	..	3.432
		218	131	44	218.07544	5.752	4.252
		219	132	45	219.07747	6.952	4.182
		220	133	46	220.08086	5.592	5.042
		221	134	47	221.08301	6.832	5.062
		222	135	48	222.08674	5.252	5.252
		223	135	49	223.08917	6.552	5.602
		224	137	50	224.09318	4.972	..
Ra	88	219	131	43	219.07824	..	5.342
		220	132	44	220.07950	7.722	6.112
		221	133	45	221.08276	5.722	6.242
		222	134	46	222.08450	7.242	6.652
		223	135	47	223.08788	5.602	7.002
		224	136	48	224.09001	6.852	7.302
		225	137	49	225.09344	5.552	7.882
		226	138	50	226.09574	6.682	..
		227	139	51	227.09982	4.902	..
		228	140	52	228.10212	6.682	..
		229	141	53	229.10448	6.622	..
		230	142	54	230.10555	7.912	..
Ac	89	221	132	43	221.08395	..	3.692
		222	133	44	222.08692	6.012	3.982
		223	134	45	223.08860	7.302	4.042
		224	135	46	224.09147	6.112	4.552
		225	136	47	225.09322	7.232	4.932
		226	137	48	226.09651	5.692	5.072
		227	138	49	227.09845	7.042	5.432
		228	139	50	228.10206	5.372	5.902

TABLE I (*contd*)

Element	(*Z*)	(*A*)	(*N*)	(*N* − *Z*)	(*M*) m.u.	(B_n) m.m.u.	(B_p) m.m.u.
Ac	89	229	140	51	229.10366	7.382	6.602
		230	141	52	230.10722	5.422	5.402
Th	90	223	133	43	223.09036	..	4.702
		224	134	44	224.09116	8.182	5.582
		225	135	45	225.09381	6.332	5.802
		226	136	46	26.09525	7.542	6.112
		227	137	47	227.09836	5.872	6.292
		228	138	48	228.09981	7.532	6.782
		229	139	49	229.10279	6.002	7.412
		230	140	50	230.10472	7.052	7.082
		231	141	51	231.10817	5.532	7.192
		232	142	52	232.11034	6.812	..
		233	143	53	233.11382	5.502	..
		234	144	54	234.11650	6.302	..
		235	145	55	235.12037	5.112	..
Pa	91	225	134	43	225.09514	..	4.162
		226	135	44	226.09823	5.892	3.722
		227	136	45	227.09953	7.682	3.862
		228	137	46	228.10200	6.512	4.502
		229	138	47	229.10331	7.672	4.642
		230	139	48	230.10599	6.302	4.942
		231	140	49	231.10783	7.142	5.032
		232	141	50	232.11095	5.862	5.362
		223	142	51	233.11250	7.432	5.982
		234	143	52	234.11586	5.622	6.102
		235	144	53	235.11854	6.302	6.102
U	92	227	135	43	227.10166	..	4.712
		228	136	44	228.10232	8.322	5.352
		229	137	45	229.10469	6.612	5.452
		230	138	46	230.10553	8.142	5.922
		231	139	47	231.10818	6.332	5.952
		232	140	48	232.10947	7.692	6.502
		233	141	49	233.11193	6.522	7.162

TABLE I (*contd.*)

Element	(*Z*)	(*A*)	(*N*)	(*N*−*Z*)	(*M*) m.u.	(B_n) m.m.u.	(B_p). m.m.u.
		234	142	50	234.11379	7.122	6.852
		235	143	51	235.11704	5.732	6.962
		236	144	52	236.11912	6.902	7.562
		237	145	53	237.12231	5.792	..
		238	146	54	238.12493	6.362	..
		239	147	55	239.12869	5.222	..
		240	148	56	240.13101	6.662	..
Np	93	231	138	45	231.11026	..	3.412
		232	139	46	232.11236	6.882	3.962
		233	140	47	233.11322	8.122	4.392
		234	141	48	234.11568	6.522	4.392
		235	142	49	235.11723	7.432	4.702
		236	143	50	236.12017	6.042	5.012
		237	144	51	237.12158	7.572	5.682
		238	145	52	238.12514	5.422	5.312
		239	146	53	239.12730	6.822	5.772
		240	147	54	240.13002	6.262	6.812
		241	148	55	241.13250	6.502	6.652
Pu	94	232	138	44	232.11338	..	5.022
		233	139	45	233.11555	6.812	4.952
		234	140	46	234.11616	8.372	5.202
		235	141	47	235.11844	6.702	5.382
		236	142	48	236.11962	7.802	5.752
		237	143	49	237.12192	6.682	6.392
		238	144	50	238.12366	7.242	6.062
		239	145	51	239.12653	6.112	6.752
		240	146	52	240.12862	6.892	6.822
		241	147	53	241.13154	6.062	6.622
		242	148	54	242.13413	6.392	6.512
		243	149	55	243.13740	5.712	..
Am	95	237	142	47	237.12302	..	4.742
		238	143	48	238.12571	6.292	4.352
		239	144	49	239.12740	7.292	4.402

TABLE I (*contd.*)

Element	(Z)	(A)	(N)	(N−Z)	(M) m.u.	(B_n) m.m.u.	(B_p) m.m.u.
Am	95	240	145	50	240.13023	6.152	4.442
		241	146	51	241.13151	7.702	5.252
		242	147	52	242.13489	5.602	4.792
		243	148	53	243.13686	7.012	5.412
		244	149	54	244.14011	5.632	5.432
Cm	96	238	142	46	238.12713	..	4.032
		239	143	47	239.12941	6.702	4.442
		240	144	48	240.13044	7.952	5.102
		241	145	49	241.13223	7.192	6.142
		242	146	50	242.13420	7.012	5.452
		243	147	51	243.13694	6.242	6.092
		244	148	52	244.13880	7.122	6.202
		245	149	53	245.14138	6.402	6.872
Bk	97	243	146	49	243.13860	..	3.742
		244	147	50	244.14122	6.362	3.862
		245	148	51	245.14229	7.912	4.652
		246	149	52	246.14547	5.802	4.052
		249	152	55	249.15186	..	..
		250	153	56	250.15591	4.932	..
Cf	98	243	145	47	243.14131	..	..
		244	146	48	244.14211	8.182	4.632
		245	147	49	245.14368	7.412	5.682
		246	148	50	246.14543	7.232	5.002
		248	150	52	248.14920	..	..
		249	151	53	249.15180	6.382	..
		250	152	54	250.15395	6.832	..
		251	153	55	251.15725	5.682	
		252	154	56	252.15988	6.352	..
		253	155	57	253.16321	5.652	
	99	246	147	48	246.14657	..	5.252
		247	148	49	247.15011	5.442	3.462
		253	154	55	253.16296	..	5.062
Centurium	100	254	154	54	254.16570	..	5.402
		255	155	55	255.16886	6.022	..

From these values of binding energies for neutrons and protons, it is clearly seen that the binding energies for even numbers are always higher than that for odd ones. At magic numbers 82 and 126, for protons and neutrons respectively, the binding energy is maximum. These observations were also verified by drawing graphs between even or odd neutron and proton numbers and their binding energies for different $(N-Z)$ values. Also a smaller peak was obtained at 88 in the case of protons, for most of the $(N-Z)$ values. There was a gradual increase and decrease in the binding energies for neutrons and protons respectively for higher numbers.

The difference between the binding energy of a neutron, B_n in a nucleus with odd (or even) N and the average of that of its two neighbours with the same $(N-Z)$ and odd (or even) N is denoted by D_n.

$$D_n = B_n - [B_n(N+2) + B_n(N-2)]/2$$

The average of D_n for all available $(N-Z)$ values is denoted by $[D_n]_{AV}$ and plotted against N(figure 1.). Similarly $(D_p]_{AV}$ is defined and plotted against Z (figure 2.).

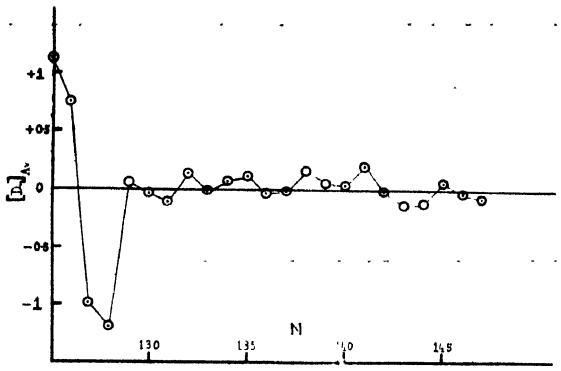

Fig. 1. Variation of $[D_n]_{AV}$ with N

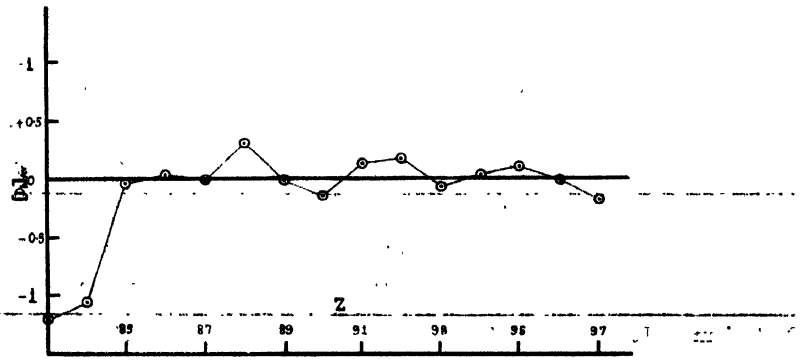

Fig. 2. Variation of $[D_p]_{AV}$ with Z.

For neutrons, the maximum $[D_n]_{AV}$ occurs at $N = 125$. It falls gradually to 126 after which the fall is very sharp. Smaller peaks were also obtained at neutron numbers 129, 132, 135, 138, 141, 145. Two adjacent smaller peaks were separated from each other by three neutron numbers, except in the case of $N = 145$.

For protons, the $[D_p]_{AV}$ curve also fluctuates. The minimum value of $[D_p]_{AV}$ occurs at $Z = 83$. Smaller peaks were observed at proton numbers 88, 92, 95. The adjacent smaller peaks are separated from each other by three proton numbers, nearly.

DISCUSSION

The binding energy of neutrons or protons reaches a high value at a magic number and rapidly falls after that. This actually happens in the case of 126 neutrons and 82 protons which are known to be magic numbers. It was pointed out by Redlich (1952) that the smaller peaks occuring at other neutron and proton numbers in '$N-B_n$', '$Z-B_p$', '$N-[D_n]_{AV}$', or '$Z-[D_p]_{AV}$' graphs might be due to these numbers being magic. At neutron numbers 11, 14, 20, 24 and 28 and at protons numbers 11, 14, 20, 28 smaller peaks were observed by him and among such numbers 14, 20 and 28 are now known to be magic numbers.

From these considerations, it is obvious that there might be magic numbers after 82 protons and 126 neutrons. Also the peak occuring at 88 protons is highest among all the smaller peaks. So this might be a magic number. But other factors are to be considered before arriving at any specific conclusion.

The periodicity in the variation of $[D_n]_{AV}$ and $[D_p]_{AV}$ is also significant and deserves considerations.

The increase in the binding energy of neutrons and protons for even N or Z values is due to their greater stability.

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